IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M06/5/MATME/SP1/ENG/TZ1/XX+

22067301

MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 3 May 2006 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

0116

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. (a) Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

(i) Write down the value of a.

(ii) Find the value of b.

(b) Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ -9 & 23 \end{pmatrix}$.

Find the value of q.

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2. Let A and B be independent events such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$.

(a) Find $\mathrm{P}(A \cap B)$.

(b) Find $\mathrm{P}(A \cup B)$.

(c) Are A and B mutually exclusive? Justify your answer.

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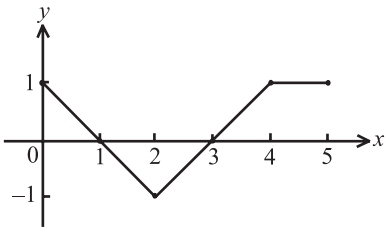
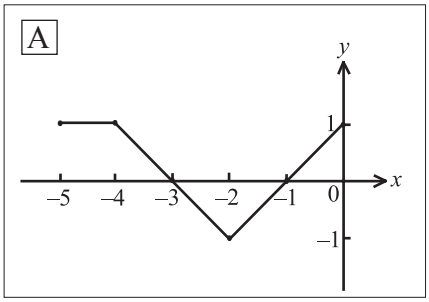
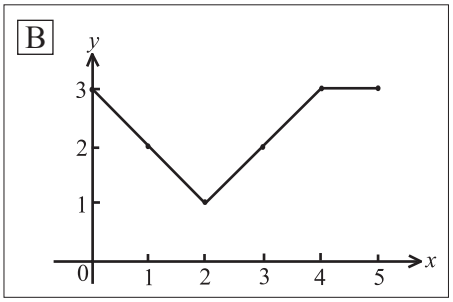
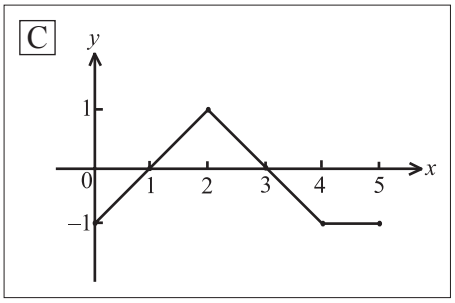
3. The following diagram shows part of the graph of $f(x)$.

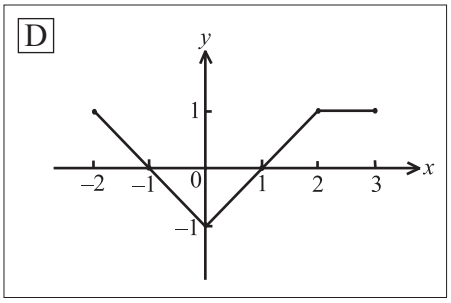


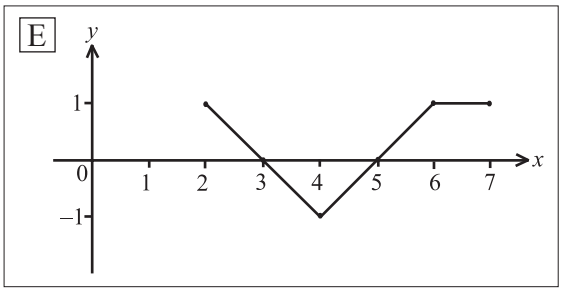
Consider the five graphs in the diagrams labelled A, B, C, D, E below.











(a) Which diagram is the graph of $f(x+2)$?

(b) Which diagram is the graph of $-f(x)$?

(c) Which diagram is the graph of $f(-x)$?

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4. The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm.

(b) In this group of students, 11.9 % have heights less than d cm. Find the value of d.

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5. (a) Let $f(x) = e^{5x}$. Write down $f'(x)$.

(b) Let $g(x) = \sin 2x$. Write down $g'(x)$.

(c) Let $h(x) = e^{5x} \sin 2x$. Find $h'(x)$.

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6. Let $f(x) = a(x-4)^2 + 8$.

(a) Write down the coordinates of the vertex of the curve of f.

(b) Given that $f(7) = -10$, find the value of a.

(c) Hence find the y-intercept of the curve of f.

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7. Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a) Find $(g \circ f)(-2)$.

(b) Find $f^{-1}(x)$.

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8. Consider the four numbers a, b, c, d with $a \leq b \leq c \leq d$, where $a, b, c, d \in \mathbb{Z}$.
The mean of the four numbers is 4.
The mode is 3.
The median is 3.
The range is 6.

Find the value of a, of b, of c and of d.

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9. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) Write down the inverse matrix $\boldsymbol{A}^{-1}$.

(b) Consider the equation $\boldsymbol{AX} = \boldsymbol{B}$.

(i) Express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$ and $\boldsymbol{B}$.

(ii) **Hence**, solve for $\boldsymbol{X}$.

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10. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

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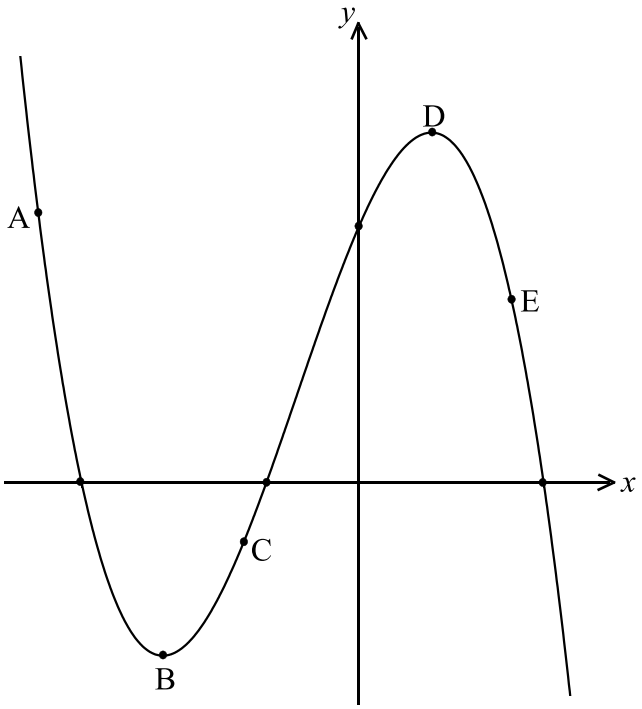
11. The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

12. The velocity, v m s^{-1}, of a moving object at time t seconds is given by $v = 4t^3 - 2t$.
When $t = 2$, the displacement, s, of the object is 8 metres.

Find an expression for s in terms of t.

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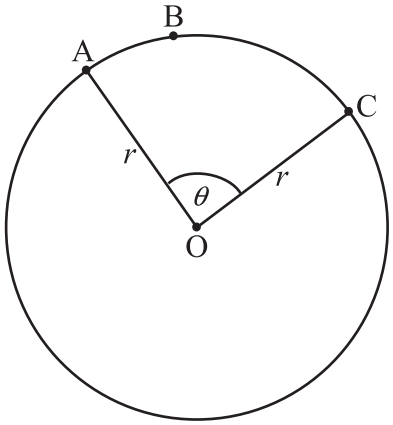
13. The following diagram shows a circle with radius r and centre O. The points A, B and C are on the circle and $\mathrm{A\hat{O}C} = \theta$.



The area of sector OABC is $\frac{4}{3}\pi$ and the length of arc ABC is $\frac{2}{3}\pi$.

Find the value of r and of θ.

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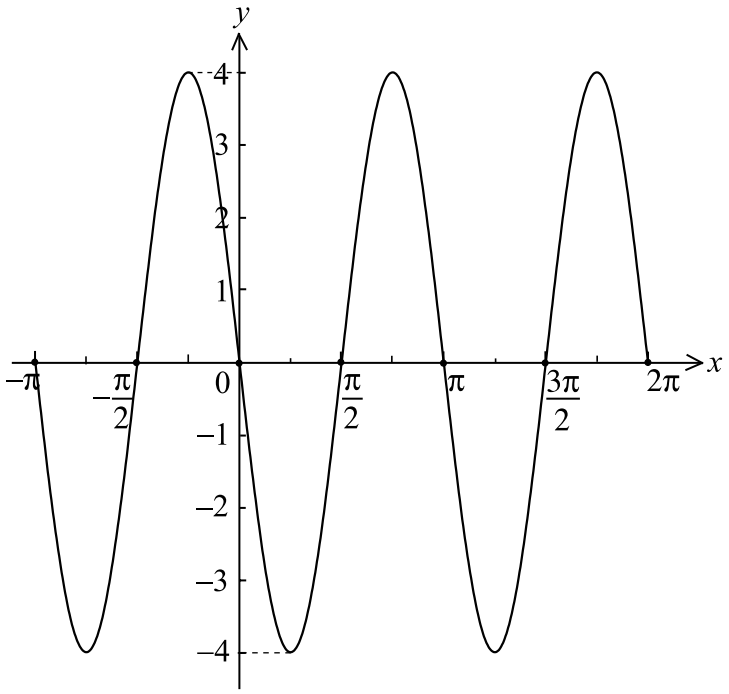
14. Let $f(x) = a\sin b(x-c)$. Part of the graph of f is given below.



Given that a, b and c are positive, find the value of a, of b and of c.

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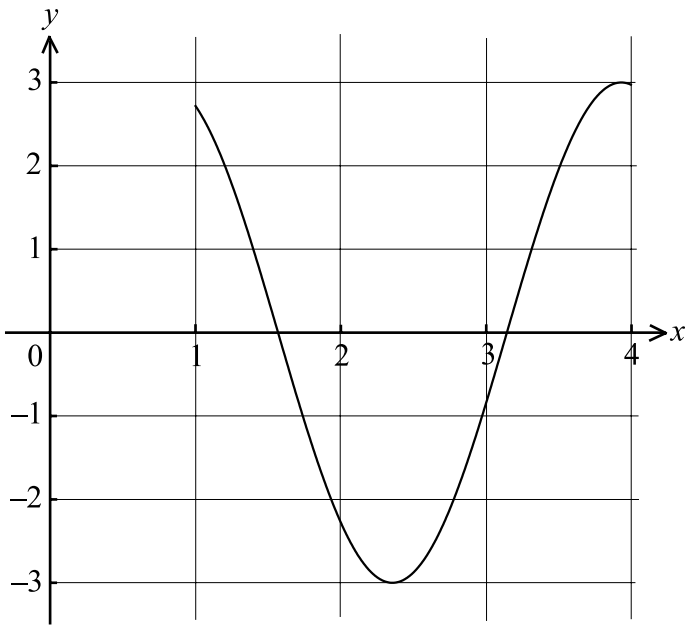
15. Let $f(x) = 3\sin 2x$, for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$ for $1 \le x \le 4$. The graph of f is shown below.



(a) On the same diagram, sketch the graph of g.

(b) One solution of $f(x) = g(x)$ is 1.89. Write down the other solution.

(c) Let $h(x) = g(x) - f(x)$. Given that $h(x) > 0$ for $p < x < q$, write down the value of p and of q.

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